

UNCONSTRAINED PLASTERING – A NEW ALL HEXAHEDRAL MESH GENERATION ALGORITHM

Matthew L. Staten^{*}, Steven J. Owen^{*} and Ted D. Blacker^{*}

^{*} Sandia National Laboratories[†], Albuquerque, NM, U.S.A

e-mail: mlstate@sandia.gov, sjowen@sandia.gov, tdblack@sandia.gov

web page: <http://www.sandia.gov>

Key words: Mesh Generation, Hexahedra, Plastering, Paving

Summary. *In the search for a reliable hexahedral meshing algorithm on arbitrary geometry assemblies, a new algorithm is being studied called Unconstrained Plastering¹. The goal is to generate conformal all-hexahedral meshes on any geometry assembly. Unconstrained Plastering is a new unproven idea. Implementation on a prototype has begun. Although the authors are optimistic, evidence of its reliability is still forthcoming.*

1 INTRODUCTION

The goal of finding an algorithm to robustly generate high quality conformal all-hexahedral meshes on any geometry assembly remains elusive. Unlike tetrahedral meshing, where reliable methods are available², the additional constraints imposed by hexahedral topology makes the hexahedral meshing much more difficult.

For quadrilateral mesh generation, Paving³ has been an effective means of generating all quad meshes on arbitrary surfaces. Paving starts by meshing the boundary curves of the surfaces, followed by the definition of fronts and front states based on angles between adjacent boundary mesh edges. Using an advancing front technique, these fronts are advanced inward using a variety of connectivity techniques until only small unmeshed voids remain at the center. These unmeshed voids are simple polygons, usually bounded by six or fewer mesh edges. Simple template meshes are inserted into these voids.

The 3D corollary to Paving is Plastering^{4,5}. Like Paving, Plastering first begins by pre-meshing the boundary. In 3D, this requires paving the boundary surfaces of the volumes. Fronts with states are defined and advanced inward using an advancing front technique similar to paving. However, the additional constraints imposed by hexahedral topology cause the Plastering algorithm to break down. The unmeshed voids that remain are typically very complex polygons with closely spaced and randomly oriented quadrilaterals. Plastering is only able to resolve these unmeshed voids on simple models.

The root cause of Plastering's inability to resolve the remaining unmeshed voids is its attempt to honor an a priori boundary quad mesh. Plastering is what is called an Outside-In

[†] Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000

The submitted manuscript has been authored by a contractor of the United States Government under contract. Accordingly, the United States Government retains a non-exclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for the United States Government purposes.

method, which meshes the outside of the volume first followed by attempting to fill the inside. Outside-In methods are attractive because they ensure the ability to mesh assemblies with conformal meshes. However, in the case of plastering, the apriori boundary quad mesh is the very cause of the algorithm's failure.

This presentation will introduce a new algorithm called Unconstrained Plastering¹, which uses an advancing front approach similar to Plastering. However, the advancing fronts advance unconstrained layers of hexahedra from an *unmeshed volume boundary*. The final boundary mesh is a result of the interior meshing process. The drawback to Unconstrained Plastering is its inability to honor a pre-defined boundary quad mesh. However, this is outweighed by the prospect of generating conformal hex meshes on any geometry assembly.

There have been other hexahedral meshing algorithms presented that also remove the constraint of an apriori boundary quad mesh^{6,7}. These methods can be classified as Inside-Out methods since they first mesh the interior of the volume, usually with an octree-grid method, which is then fitted to the boundary. However, these Inside-Out methods give no ability to create conformal meshes on assemblies. Unconstrained Plastering is able to generate conformal meshes on assemblies provided the entire assembly is meshed at once.

2 UNCONSTRAINED MESH GENERATION

As mentioned previously, implementation on a full 3D prototype of Unconstrained Plastering has begun. This abstract introduces the concepts with a 2D example. The conference presentation will extend the 2D algorithm to 3D.

2.1 Unconstrained Mesh Generation in 2D (Unconstrained Paving)

Figure 1 through Figure 12 illustrate a simple surface being meshed with Unconstrained Paving. Unlike traditional paving, the boundary curves have not been pre-meshed in Figure 1. Figure 2 shows the advancement of a single unconstrained row. Since the boundary is not meshed, this row is unconstrained. In other words, this row is free to have as many quadrilateral elements as desired. In Figure 3 an additional unconstrained row has been advanced. Since this additional row crosses the previously defined row, a single quadrilateral element is formed where the 2 rows cross. Additional unconstrained rows are advanced in Figure 4 through Figure 9. Only where 2 rows cross is a quadrilateral element formed. At this point, the various regions of the surface can be categorized as either quad elements (shaded gray, where 2 rows have crossed), unmeshed void (patterned, where no rows have crossed), or connecting tubes (white space, where one row has crossed). The connecting tubes connect the unmeshed void to the boundary such that the unmeshed void is free to be meshed with as many divisions along any of its edges, leaving it completely unconstrained. Figure 5 and Figure 6 show what happens when opposing fronts collide. The process continues until in Figure 9 the unmeshed voids are approximately 1-2 times the desired element size. At this point, the connecting tubes are analyzed and refined until they are of appropriate size as shown in Figure 10. Each unmeshed void is then resolved by splitting one of the adjacent connecting tubes and collapsing the unmeshed void. In the general case Midpoint Subdivision⁸ can be used to resolve these unmeshed voids.

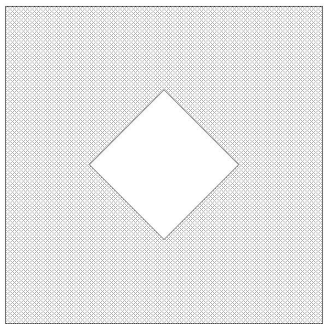


Figure 1

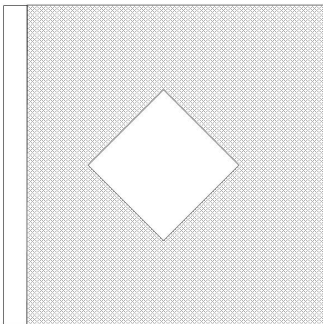


Figure 2

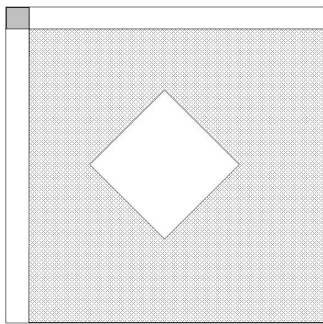


Figure 3

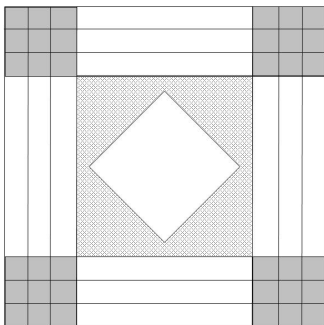


Figure 4

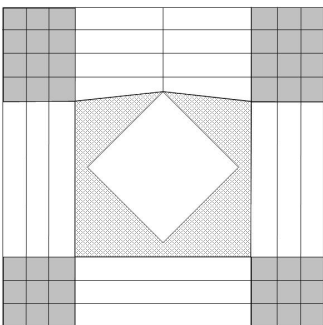


Figure 5

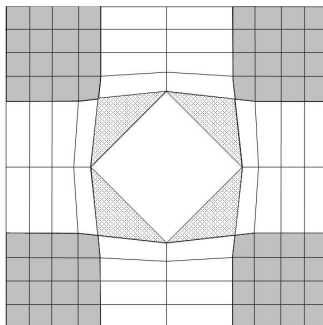


Figure 6

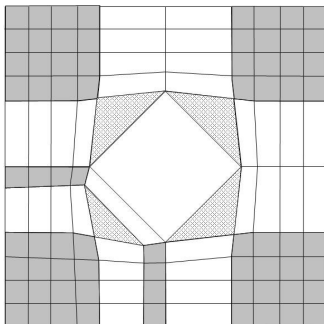


Figure 7

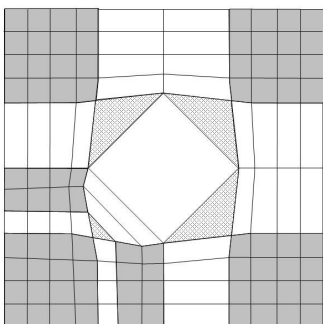


Figure 8

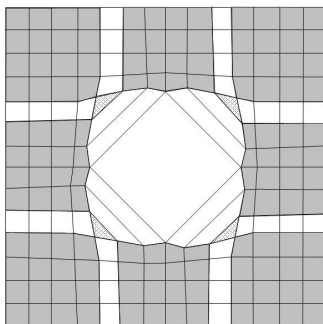


Figure 9

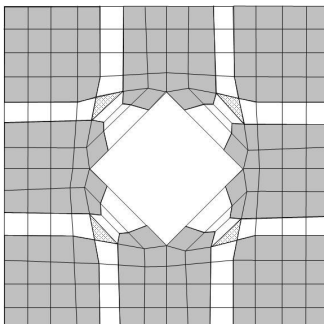


Figure 10

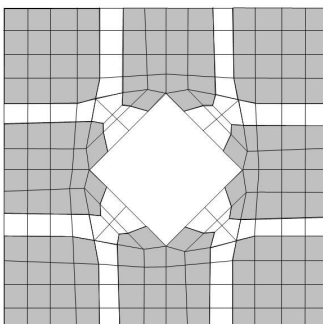


Figure 11

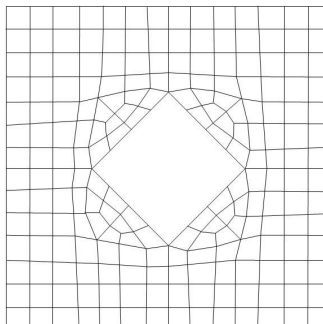


Figure 12

2.2 Unconstrained Mesh Generation in 3D (Unconstrained Plastering)

In 3D, Unconstrained Plastering advances unconstrained layers of hexahedra. Before any intersections with other layers, the number, size, and orientation of the hexahedra in a layer is left unconstrained. When two unconstrained layers cross, an unconstrained column of hexahedra is formed, which fixes the orientation of the hexahedra in this column, however, the size and number is left unconstrained. Only when 3 unconstrained layers cross is the size and orientation of a single hexahedral element fixed. In the general case, the unmeshed voids are resolved using Midpoint Subdivision⁸. Figure 13 shows a simple solid meshed with Unconstrained Plastering

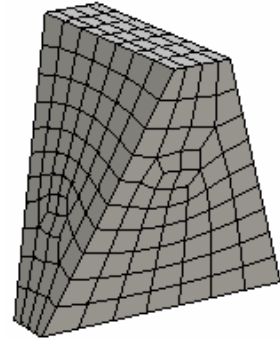


Figure 13 Simple solid meshed with Unconstrained Plastering

3 CONCLUSIONS

The concept of meshing surfaces and volumes by advancing unconstrained layers of quads or hexahedra has been introduced. More details will be provided in the conference presentation. Although valuable as a thought experiment, implementation of Unconstrained Paving is not planned since traditional paving meets quadrilateral meshing requirements. However, implementation has begun on a full 3D prototype of Unconstrained Plastering. Once implementation progresses, the flexibility and robustness of Unconstrained Plastering will be determined. Subsequent publications will present additional details as the research matures.

REFERENCES

- [1] M. L. Staten, S. J. Owen and T. D. Blacker, "Unconstrained Paving & Plastering: A New Idea for All Hexahedral Mesh Generation", *Proceedings. 14th International Meshing Roundtable* (2005).
- [2] P.L. George, and H. Borouchaki. "Delaunay Triangulation and Meshing: Application to Finite Elements", © Editions HERMES, Paris, 1998.
- [3] T. D. Blacker and M. B. Stephenson, "Paving: A New Approach to Automated Quadrilateral Mesh Generation", *International. Journal for Numerical. Methods in. Engineering* , **32**, 811-847 (1991).
- [4] S. A. Canann, "Plastering: A New Approach to Automated 3-D Hexahedral Mesh Generation", *American Institute of Aeronautics and Astronautics* (1992).
- [5] T. D. Blacker and R. J. Meyers, "Seams and Wedges in Plastering: A 3D Hexahedral Mesh Generation Algorithm", *Engineering with Computers*, **2**, 83-93 (1993).
- [6] R. Schneiders, "A Grid-based Algorithm for the Generation of Hexahedral Element Meshes", *Engineering with Computers*, **12**: 168-177 (1996).
- [7] G. D. Dhondt, "Unstructured 20-Node Brick Element Meshing", *Proc. 8th International Meshing Roundtable*, 369-376 (1999).
- [8] T. S. Li, R. M. McKeag, and C. G. Armstrong, "Hexahedral Meshing Using Midpoint Subdivision and Integer Programming", *Computer Methods in Applied Mechanics and Engineering*, **124**, 1-2, 171-193 (1995).